Secondary School Students' Understanding Of Inequalities In A Linear Programming Task

Kevin M. White, CBC St Kilda

A two-part study of the difficulties experienced by senior secondary (Year 12) students in writing inequalities from graphical linear programming problem statements is reported. The first part of the study identifies possible difficulties and the second part evaluates a teaching programme designed to alleviate these difficulties. The teaching programme, based around a set of specific heuristics, is successful in alleviating some of these difficulties. The "pronumeral-as-object" notion and certain misconceptions of "constraint" prove resistant to remediation.

For many students, the last year of secondary education, Year 12 in Australia, is the pinnacle of their schooling. The widely-held perception in the general community of the value of mathematics in developing thinking skills and the fact that some form of Year 12 mathematics is pre-requisite for a range of tertiary courses lead a very substantial number of students to study some mathematics course in Year 12. By this stage, it is assumed, students possess an understanding of mathematical concepts and processes sufficient for them to cope with the demands of courses offered at this level.

As their teachers well know, however, not all students who present themselves for Year 12 courses have in fact achieved such mathematical understanding. Documented examples of common difficulties include: i) lack of understanding of algebraic symbols and their manipulation, ii) lack of understanding of relationships between variables, iii) difficulties in graphing relationships between variables, and iv) difficulties in interpreting worded statements or "problems".

Herscovics (1989) believed that learning difficulties were of two basic types:

1. The learner attempts to map new material onto an existing mental structure which is valid in another domain but inappropriate for the knowledge to be learned. Difficulties i) to iii) above might belong to this category.
2. The inherent structure of the new material might be such that the learner has no existing mental structure which would allow assimilation of the new material. Difficulty iv) above might be of this type.

Herscovics coined the term "*cognitive obstacle*" (1989, p. 61) to refer to either 1. the existing mental structure (or its attempted use) or 2. the structure of the new material.

This report focuses on the cognitive obstacles to the writing of inequalities from worded statements experienced by two groups of Year 12 students. The context is that of linear programming, one of the more demanding topics from the current Victorian Certificate of Education (VCE) Further Mathematics Units 3 and 4 course, regarded as the least difficult of the three VCE pre-tertiary mathematics subjects offered at this level. The material from which this report is drawn is a study by the author (White, 1995) of the cognitive obstacles experienced by students in learning to solve by graphical means linear programming problems involving two decision variables.

An example of the type of problem considered in this study is the following.

> A small toy factory produces models of cars and boats. There is sufficient plastic to produce 12 models a day. (At least) Five cars and three boats a day are ordered. The profit on cars is \$1.00 and on boats is \$1.50. ... Find the possibility which gives the maximum profit (Andrews, 1990, p. 210).

The students must first read this densely worded statement and interpret it. They must understand a number of inter-related concepts, both practical and mathematical, and be able to express these in various representations (verbal, symbolic, graphical). Some of the mathematical concepts are specific to linear programming and would be taught as part of a linear programming unit. Other concepts assumed by the syllabus to have been gained previously by the students would include the notions of "variable", "equality", "inequality" and "the co-ordinate plane". This study will examine students' understanding of inequalities via their performance on linear programming tasks.

Literature Review

Although there have been many research studies of cognitive obstacles related to the writing of equalities from worded statements, the area of writing inequalities seems to have received scant attention thus far. Research on the former which is relevant to this study is summarized below.

Obtaining an Equation from a Worded Description

Students commonly find it difficult to obtain an equation from a verbal description of a relationship between two or more variables. The classic case of this in the mathematics education literature is the "Student-Professor Problem", as follows.

> Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university". Use S for the number of students and P for the number of professors (Rosnick and Clement, 1980, p. 4).

Only 63% of first-year engineering students at a major United States university could answer this problem correctly. Two-thirds of the errors took the form of a "reversed equation" ("6S = P" instead of "S = 6P"). Rosnick and Clement attempted remediation through tutoring interviews but concluded that "the misconceptions students possess relating to variable and equation are deep seated and resistant to change" (p. 23).

The researchers attempted to determine the particular "misconceptions" held by the students, or, in the terminology of this study, the "cognitive obstacles" concerned. From interview protocols, two conceptual sources of reversal error were identified (Clement, Lochhead & Monk, 1981; Clement, 1982):

In *syntactic word order matching*, the student has the notion that a direct mapping of the key words of the problem statement into algebraic symbols will produce the required equation, thus obtaining "6S = P".

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There are six times as many students as professors.
          6                 S       =       P
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In *semantic static comparison*, the student was believed to have formed a mental picture of the problem. "There are six times as many students as professors" implies that the group of students is much larger than the group of professors. Some subjects of the study made this hypothetical mental picture concrete by means of a diagram.

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S  S  S  S  S  S              P
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These subjects then represented the relationship by the equation "6S = P", in which "6S" is used to show the larger group and "P" is used to show the smaller group. Other explanations of the reversal error have been offered, among them MacGregor and Stacey's (1993) theory of *intuitive cognitive models.* The authors studied the attempts of over 1000 Australian secondary school children at formulating linear equations involving two variables. Although the test items were designed to eliminate all causes of reversal error given previously in the literature, a high incidence of reversal error was observed. It was postulated that the reversed equations were attempts to represent on paper cognitive models of the relationship between variables. Because the models were based on the notion of comparison, rather than equality, they could not be translated directly into mathematical notation. Clement's (1982) static comparison model was regarded as a special case of such an intuitive cognitive model when concrete referents, e.g., "students" and "professors", were involved (MacGregor and Stacey, 1993, p. 229). The consequence of this hypothesis which is relevant to the present study is that relationships between variables ought to be paraphrased or re-organized in some fashion before an attempt is made to express them mathematically.

Difficulties in Problem Comprehension

De Corte and Somers (1981) obtained evidence that the difficulties experienced by sixth grade children in solving word problems were due mainly to their inability to master the comprehension phase, not the solution phase, of mathematical problem solving. MacGregor (1989, 1991) proposed that the difficulty of interpreting mathematical language stems from the fact that it demands an awareness of structure not normally required in natural-language processing. The importance of understanding problem structure was confirmed by Low and Over (1989), who found that the capacity of Year 10 students to identify necessary and sufficient data for solution of algebraic story problems accounted for 90% of variance in the solution rates of these problems.

Students' Understanding of Equalities and Inequalities

MacGregor and Stacey (1993) noted that some students, in attempting to represent relational statements mathematically, used inequality symbols when an "=" sign was required, so it might be concluded that students' understanding of both equalities and inequalities is likely to be deficient.

The discussion of possible difficulties related to the writing of inequalities and the learning of linear programming prompts the following questions.

1. What cognitive obstacles in the area of inequalities are possessed by the students of this teacher/ researcher?
2. In what manner and to what degree do these cognitive obstacles interfere with the learning of graphical linear programming?
3. How effectively does the teaching process deal with these cognitive obstacles?

Methodology

The general methodology used for the investigation was that of "action research", whose primary goal, the solution of a local problem in a local setting (Gay, 1992), is consistent with the stated purpose of the study. A suitable teaching programme was developed and evaluated by descriptive means, including analysis of teacher–student interactions in the actual learning situation and of students' responses to test items. A key feature of action research which was crucial to this study was the involvement of the researcher as teacher. A limitation of this research model is that no method of control typical of other research approaches was used, nor was it intended that the results ought to be generalizable to any other setting (Gay, 1992, p. 11). One complementary advantage of the action research model was the freedom of the teacher-researcher to respond to the perceived demands of the moment, in the desire to effect (possibly immediate) improvement in the understanding of his students.

The permission of the school Principal for the teacher to conduct this investigation was sought and obtained, on condition that the demands of the syllabus be met and that the procedure followed would be consistent with the normal teaching pattern. This reasonable condition placed restrictions on the time, teaching approaches and research methods available for the classroom part of the investigation, e.g., use of controlled comparison studies was clearly inappropriate. One teaching method considered by the teacher was that of group work but this was not adopted because the students had little experience of this in mathematical contexts and there was insufficient time to foster suitable dynamics. The permission of the students for the teacher to audiotape the lessons and use the transcripts for research was sought and obtained. In any reference to an individual student which follows, the name has been altered.

The subjects of the study were students in two classes of at least 20 taught by the author (1993 & 1994) at an inner Melbourne, Catholic, boys' senior secondary college. The study was in two parts: the first part, the "trial" unit of 1993, aimed to determine difficulties students had in learning linear programming; the second part, the teaching unit of 1994, was to evaluate a programme designed to alleviate these difficulties. The basic design followed in each unit was that of pre-test, teaching/learning and post-test. The teaching/learning time for each unit was 5 lessons of approximately 45-50 minutes.

The "Trial" Unit (1993)

The "trial" unit of 1993 was an exploratory research tool in which the researcher taught linear programming to a group of students similar to that which would be the subject of the 1994 teaching model. The purpose was to identify the cognitive obstacles to the learning of linear programming possessed by the students in order to formulate a suitable teaching program for this topic. Only those cognitive obstacles considered most relevant to the writing of inequality statements will be discussed here.

In the teaching/learning process, some students found it difficult to express mathematically inequality terms, e.g., "not more than". The transcripts (given in their entirety in White, 1995) showed that it took four students to attempt a mathematical description of the statement, "Angela can't buy more than 3 packets of crisps", before a correct answer, "c is less than or equal to 3," was obtained. In the post-test, some students experienced similar difficulties in trying to write mathematically, "at least".

Another cognitive obstacle to the writing of inequality statements was the notion that the pronumeral represented an object. When the class was considering the problem of expressing mathematically, "The number of cars is at least 5", and Josh mentioned "5c", the teacher reminded the students that "c" stood for "the number of cars" and asked what "5c" would represent. When Jim replied, "5 cars", the teacher repeated the meaning of "c". Jim insisted that "5c" was "still 5 cars". When the class returned to the original problem, Jack stated, "5 times c equals the number you're going to get".

The responses to one of the pre-test items suggested that half the class had a good intuitive understanding of "constraint", yet 14 out of 19 students answered incorrectly a post-test item which required them to express a constraint mathematically. Four incorrect responses could *not* be explained by a "pronumeral-as-object" conception.

Thus the trial unit suggested these (probably inter-related) cognitive obstacles to the writing of inequality statements.

Cognitive Obstacle 1: The lack of understanding of terms of inequality, e.g., "at least".

Cognitive Obstacle 2: The notion that algebraic letters represent concrete objects, e.g., "5c" means "5 cars".

Cognitive Obstacle 3: The difficulty of writing constraints in mathematical language.

The Teaching Unit (1994)

Choice and Justification of Strategies for Teaching

Following the advice of Bell (1991) and Sweller and Low (1992), the unit focused on several key examples, whose solution was to be developed through the medium of class discussion. A set of specific heuristics (Steps 1-8 in Table 1) provided a model for solution. It was hoped that these heuristics would help break the problem down into manageable steps and avoid or alleviate cognitive obstacles to linear programming.

Table 1: Summary of Steps in Solving Graphically a Linear Programming Problem

Step 1:	Locate the "decision variables".
Step 2:	Name the decision variables, representing each by a different letter (usually x or y).
Step 3:	Name the variable which must be maximized or minimized (e.g., profit, or cost) and express it in terms of x and y, the decision variables.
Step 4:	What constraints (restrictions) are imposed on each of the decision variables? State these in words using "The number of ...".
Step 5:	Express "The number of ..." constraints in mathematical language, using inequality symbols.
Step 6:	Using x and y axes, sketch the areas defined by the inequality statements. Hence find the "feasible region".
Step 7:	Find the co-ordinates of the vertices of the feasible region.
Step 8:	Find the solution to the problem by calculating the profit (or cost, etc.) for each of the vertices of the feasible region.

Some justification is given here for those steps which were intended to alleviate Cognitive Obstacles 1-3 to the writing of inequality statements.

Cognitive Obstacle 1: The student was asked to "translate" inequality terms, e.g., "no more than" or "at least", into "less than or equal to" or "greater than or equal to", each of which has a mathematical representation ("≤" or "≥") known – hopefully – to the student, who was to verify the accuracy of his "translation" by examining possible numerical values of the variable concerned. This formed part of Steps 4-5.

Cognitive Obstacle 2: Once the key or "decision" variables had been identified (Step 1), the student was to write a statement defining those variables: "Let the number of boats produced per day be x" (Step 2). This follows MacGregor (1986), who suggested using "The number of" statements to introduce the use of the literal symbol as representing a variable. The symbols for the decision variables were "x" and "y", not the initial letters of the objects themselves, in order to avoid confusion with common shorthand, "b = boats" or "c = cars", and because the students had some experience in using "x" and "y" as numbers and in graphing relations involving "x" and "y".

Cognitive Obstacle 3: Step 4 required the student to write a "The number of" statement, such as, "The number of boats produced per day is greater than or equal to 3", to express each constraint on the decision variables. This form would enable a "syntactic" translation (Step 5) to "x ≥ 3". The re-structuring of constraint statements is consistent with MacGregor and Stacey's (1993) conclusion that relationships between variables need to be re-organized before being expressed mathematically.

Teaching/learning

Lesson 1 focused on the notion of "constraint" and its mathematical expression as an inequality. Consideration of a pre-test item which required an intuitive understanding of the idea of constraint led naturally to the testing of each alternative by the device of numerical comparison, a strategy readily understood by the students.

> If Optus Affirmative can produce no more than 12 litres of orange juice and no more than 20 litres of ginger beer per day, which of the following daily sales are possible?
> 1: 6 litres of orange juice and 12 litres of ginger beer ....
> T. Ah, Pietro, what would you say about number 1?
> Pietro It is possible because he can produce 6 litres of orange juice; he can produce 12 litres ...

The word "constraint" was introduced in the following fashion.

> T. ... Now a constraint is a restriction on something; ... with the orange juice, it says here you can produce no more than 12 litres. What's the least the person can sell?
> x. Nothing.
> T. Nothing. O.K. So the least for the orange juice is 0 litres. What's the most able to be sold for orange juice?
> x. 12. ...
> Justin That's the maximum.
> T. That's the maximum. Right, very good. So the minimum is 0 litres and the maximum is 12. Anywhere in between 0 and 12 is able to be sold ... this is the restriction on the orange juice.

This approach appeared to be successful. The teacher used the word "restriction" rather than "limit" because "limit", in general usage, refers to a maximum only. In linear programming problems, a minimum is often considered. Nor do constraints always give simple maxima or minima, e.g., "At least twice as many cars as boats must be made."

Lesson 3 introduced the students to Steps 4-8 (Table 1). The intention of the teacher was to re-write the statement, "At least three boats are ordered", in the form, "The number of boats is greater than or equal to three", thus permitting a syntactic translation to the inequation, "x ≥ 3" (Step 5). Some students initially had difficulty expressing "The number of boats has a minimum of three" in the desired form.

T. Now how can we express that in language ... "greater than" or "less than" or "greater than or equal to" or "less than or equal to"? What can we say about the number of boats? Adrian?
Adrian Less than or equal to. ...
Riccardo Greater than. ...
Terry ... x is greater than or equal to.

The teacher then used the strategy of verifying the suitability of possible statements by examining different values for the number of boats, a strategy which appeared to be successful in helping the students to express the constraints on the decision variables.

T. Hold on, which — We have to decide which one it is. We have to make "at least three boats". So is 3 boats O.K.?
xx. Yes.
T. Is 4 boats O.K.?
Adrian Yes. Anything more than —
T. Right, so is 2 boats O.K.?
xx. No.
T. Why not?
Adrian Because 3 boats have to be bought by someone, by 3 people.
T. That's right. So we need to make at least that number.
Anthony So that's greater than.
T. Yes, Anthony. It's greater than or equal to 3. ...The number of boats is greater than or equal to 3.

The students then found it easy to write the inequality, "$x \geq 3$", having obtained the worded statement, "The number of boats is greater than or equal to 3".

The Post-test of the 1994 Unit

Responses to selected items on the post-test of the 1994 unit are examined in order to gain insight into students' notions of constraints and inequalities.

Item B1: "At the local milk bar, Cherry Ripes are $1 and a dozen eggs cost $2. If I have $10 to spend, write down the constraint on my spending, using x = the number of Cherry Ripes bought and y = the number of dozens of eggs bought."

Only one response to Item B1 was completely correct. There were, however, common notions of "constraint". A correct notion was that a constraint involves an inequality: nine of the 21 responses involved at least one inequality. One incorrect notion was that a constraint is any permissible solution for the variables concerned: six of the 21 responses were of this type. A possible source of this in the teaching process was the testing of possibilities for x and y to verify that a chosen inequality was correct. However, the word "constraint" was used only in relation to "inequality" or "restriction" or "limit", each of which appears to contradict the previous notion. Another incorrect notion was that a constraint is the maximum value of the variable: there were two such responses. A possible explanation for the notion that an equality, as in, "$4x + 3y = 10$", describes a constraint is that some students attach their own meanings to mathematical symbols. If this response meant "4 Cherry Ripes and 3 dozen eggs cost $10", it would make sense, and, although it would not a be constraint, it would give a possible combination of x and y based on the constraint in the problem statement.

Item B2: "If x = the number of Cherry Ripes I buy, and I need to buy at least two Cherry Ripes, write down this last piece of information in mathematical language."

Item B2 (six correct answers from 21) was better answered than Item B1. The response "$x = \geq 2$" [sic!] showed confusion about the meaning of the signs for equality and inequality. The response "$x \leq 2$" used the wrong inequality sign. Whether this was due to the students' not understanding "at least" or to their not knowing the correct symbol for "greater than" cannot be known. The response "$x = 2$, $y = 4$" seemed to equate "at least two" with "equal to two". This response and "$x = 10$" maximized the total cost, suggesting that their authors confused the notions of "constraint" and "maximizing cost". The responses "$2x \leq 10$" and "$2x + y \leq 10$" seemed to involve incorrect notions of "x", possibly as the object "Cherry Ripes" or its cost. Thus incorrect answers to the same question could have their source in different cognitive obstacles.

Item B8, Step 4: "What constraints (restrictions) are imposed on each of the decision variables? State these in words using 'The number of ...'."

Three of the 21 students correctly identified the constraints and expressed in them in the desired format, "The number of ... ". Two of the these three scored the highest two marks on Item B8 (a complete linear programming problem). The post-test of 1993 gave evidence that understanding the problem constraints and expressing them in mathematical language was for most students a stumbling block. The 1994 results were similar. Some students understood the constraints but did not format them correctly; others identified the constraints but merely paraphrased them. Some did not attempt Step 4. In the teaching/learning process, this was perhaps the most difficult step, so these results were no surprise. They emphasize the persistence of Cognitive Obstacle 3.

Item B8, Step 5: "Express 'The number of ...' constraints in mathematical language, using inequality symbols."

The success rate for Step 5 of B8 was similar to that for Step 4. Each of the five students who achieved substantial success in Step 4 was able to write at least one correct inequation for Step 5. This suggests that writing "The number of... " statements to represent the problem did not hinder, and probably helped, the process of expressing these constraints mathematically. The fact that two of the four students who were successful in Step 5 either did not complete Step 4 or merely paraphrased the problem constraints prior to Step 5 demonstrates that success in Step 4 was not essential for success in Step 5. In other words, a re-structured, written version of the problem constraints may assist the student to write these constraints mathematically but it is not a pre-requisite. It cannot be concluded on this evidence, however, that the student does not enter a process by which an intermediate set of statements is constructed mentally.

Discussion

The evidence of the post-test and transcripts was that the students as a group seemed to have few difficulties understanding inequality terms, e.g., "at least" (Cognitive Obstacle 1). How much this was due to the taught technique of checking inequality statements using numerical values for variables cannot be determined.

Evidence was obtained that students who held Cognitive Obstacle 2 (the notion that literal symbols represent objects) found it difficult to write correct inequality statements. This cognitive obstacle was more obvious in the post-test responses than during the teaching/learning process. The pattern of results in the post-tests of 1994 and 1993 suggested a positive association between the ability to regard variables as numbers and success in solving the type of linear programming problems encountered.

The importance to the solution of the linear programming problem of overcoming the persistent Cognitive Obstacle 3 (difficulty in expressing constraints mathematically) was demonstrated by the post-test results. As an explanation of the existence of this obstacle for some students, these misconceptions of "constraint" were identified.

- A constraint is any permissible solution (or set of solutions) for a variable.
- A constraint is the maximum possible value of a variable.
- A constraint is expressed mathematically as an equality statement.

Students who wrote a "The number of" statement as a means of re-structuring the problem constraints were successful in formulating the inequations representing these constraints. The heuristic of using "The number of" statements is therefore not rejected. However, since few students were able to complete this step, it is suggested that some further techniques are necessary to help students interpret and represent the key features of constraint statements and other parts of linear programming problems.

Implications

It is suggested that the teaching approach adopted is worthwhile pursuing. It is noted particularly that those students who followed the set of heuristics (Steps 1 to 8) achieved a high degree of success in the solution of the linear programming problems encountered. The perhaps disappointing results of the other students might be ascribed more to their lack of confidence in tackling individual steps, rather than the use of the set of heuristics in the first place. Observing the tendency of heuristic teaching methods to lead to equivocal results, Schoenfeld (1985) offered this explanation: "Although heuristic strategies can serve as guides to relatively unfamiliar domains, they do not replace subject matter knowledge or compensate easily for its absence" (p. 73).

It is recommended that students receive more opportunities and guidance in developing skills in problem comprehension and representation, as well as in problem solution. This aim ought to lie at the heart of the school mathematics curriculum.

Further investigation, especially via interviews of individual students, of the cognitive obstacles discovered, might yield insights into students' cognitive structures and therefore the best means of alleviating these obstacles. Controlled comparison studies of the effectiveness of different teaching strategies, including Steps 1 to 8 from this study, could provide useful data to assist teachers. Given that one of the students' major difficulties in writing inequalities and in linear programming overall is likely to be problem comprehension, research into the effectiveness of different means of enhancing students' ability to represent and re-structure problems would be valuable.

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